Dynamics of a spontaneously bent ferromagnetic filament

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The performance of a ferromagnetic swimmer driven by a pulse-like magnetic field depends on its spontaneous curvature. It determines the direction of swimming and its velocity. A static configuration of the filament may be found by considering its energy, which includes the elastic energy of the filament with spontaneous curvature c_0 and bending modulus A as well as the interaction energy with the applied field. The solution for the equilibrium shape is obtained in terms of elliptic functions and reads (ϑ is the tangent $\vec{t}$ angle with the applied field, M is the magnetization per unit length, $\vec{M}$=-M$\vec{t}$, arclength is scaled by the length of a filament L):

$$sin\,sin\,\left(\frac{\vartheta}{2}\right)=sin\,sin\,\left(\frac{\vartheta_0}{2}\right)sn(\sqrt{Cm}\ (l-\tfrac{1}{2}))\ ,$$

where $Cm=\frac{MHL^2}{A}$ is the magnetoelastic number. The tangent angle on the tip of filament ϑ_0 is found from the conditions at the unclamped ends $\frac{d\vartheta}{dl}=c_0$. The shape of filament is also calculated numerically by the algorithm described in and generalized by accounting for a spontaneous curvature. In dynamics simulations time is scaled by the characteristic elastic deformation time $\zeta_\perp L^4/A$. The equation of motion of the filament is discretized for $p+1$ markers where according to the boundary conditions on the unclamped ends $\left(\frac{d^2x}{dl^2}(1),\frac{d^2y}{dl^2}(1)\right)=\left(-c_0\frac{dy}{dl}(1),c_0\frac{dx}{dl}(1)\right)$ the forces are added $(F_x^s(2),F_y^s(2))=\left(c_0\frac{(4y_2-y_3-3y_1)}{2h^2},-c_0\frac{(4x_2-x_3-3x_1)}{2h^2}\right)$ and $\vec{F}(1)=-\vec{F}(2)$ and similarly for the other end (h is the mesh size). The curvature of the filament at $l=1/2$, which is obtained by reaching an equilibrium configuration in dynamics simulations (solid circles) in dependence on Cm is shown in Fig.1. It shows good agreement with the theoretical solution (solid line).

Self-propelling motion is studied numerically. To mitigate the instability of the curved shape the pulse-like magnetic field is used. Comparison of the dynamics at $c_0=0$ and $c_0=0.5$ is shown.. It may be noticed that the bending stage (Δx decreases) proceeds slower than the relaxation stage (Δx increases). Due to this symmetry breaking self-propulsion takes place.